Statistics Notes: Numerical Examples of Cronbach's Alpha

Fergyanto E. Gunawan

June 11, 2013

Cronbach's alpha is easy to use to measure consistency of responses. For an example, we may have two variables that should measure the same thing; therefore, those two variables should be correlated. In medical, we can use two variables—ability to walk and ability to climb stairs—to measure impairment of patients having cervical myelopathy. Theoretically, we understand that the impairment will equally affect the both ability to walk and to climb. Thus, we expect a high Cronbach's alpha for the both variables.

The Cronbach's alpha for a quantity, which is a sum of k variables: $y = x_1 + x_2 + \cdots + x_k$ is defined as

$$\alpha = \frac{k}{k-1}\left(1 - \frac{\sum \sigma_{x_i}^2}{\sigma_y^2}\right), \qquad (1)$$

where σ_y^2 and $\sigma_{x_i}^2$ are the variance of y and x_i.

Now, we consider data gathered in Table 1 for variables $x_1, \ldots, x_4$. The data for variable x_2 exactly equal to x_1; data x_3 are opposite of x_1; and, data x_4 are random.

Table 1: Example Data

	Variables			
	x_1	x_2	x_3	x_4
Question 1	1	1	7	7
Question 2	2	2	6	3
Question 3	3	3	5	2
Question 4	4	4	4	7
Question 5	5	5	3	1
Question 6	6	6	2	1
Question 7	7	7	1	3

Using Eq. 1, we obtain α for combinations of variables (x_1, x_2), (x_1, x_3), and (x_1, x_4).

Table 2: Calculated αs for combinations of variables (x_1, x_2), (x_1, x_3), and (x_1, x_4)

α for variables x_1 and x_2 :	1
α for variables x_1 and x_3 :	$-\infty$
α for variables x_1 and x_4 :	-2.01

The results show that when responses of two variables are exactly the same, we obtain $\alpha = 1$. If the two variables are positively correlated, we expect a high $\alpha < 1$. In the second case, x_3 has an exact but negative correlation, we obtain $\alpha = -\infty$. Finally, when the data have no relation, we obtain a negative $\alpha = -2.01$.