Interpolation

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March 19, 2014

Introduction

These slides provides the most important aspects of the lecturing materials for the course K0572 Numerical Methods for the session of Numerical Interpolation.

Interpolation Methods

- Lagrange's method
- Newton's method

The Newton's method is computationally more superior than the Lagrange's method.

We assume: we have $n + 1$ pairs of data $\{x_i, y_i\}$

Lagrange's Method

Interpolation equation:

$$P_n(x) = \sum_{i=0}^{n} y_i \, l_i(x)$$

$$l_i(x) = \prod_{j=0, j \neq i}^{n} \frac{x - x_j}{x_i - x_j}$$

- n – The order of the polynomial, for the linear function, $n = 1$
- y_i – The y-coordinate of the data
- l_i – The basis or cardinal function
- i – The index where $i \in (0, \ldots, n)$

Lagrange's Method

The Basis Functions $l_i(x)$ for the Linear Case

$n = 1$

$$
\begin{aligned}
l_0(x) &= \frac{x - x_1}{x_0 - x_1} \\
l_1(x) &= \frac{x - x_0}{x_1 - x_0}
\end{aligned}
$$

Lagrange's Method

The Basis Functions $l_i(x)$ for the Quadratic Case

$n = 2$

$$
\begin{aligned}
l_0(x) &= \frac{x - x_1}{x_0 - x_1} \cdot \frac{x - x_2}{x_0 - x_2} \\
l_1(x) &= \frac{x - x_0}{x_1 - x_0} \cdot \frac{x - x_2}{x_1 - x_2} \\
l_2(x) &= \frac{x - x_0}{x_2 - x_0} \cdot \frac{x - x_1}{x_2 - x_1}
\end{aligned}
$$

Newton's Method

Interpolation equation:

$$\begin{aligned} P_n(x) &= a_0 + (x - x_0)a_1 + (x - x_0)(x - x_1)a_2 + \cdots \\ &+ (x - x_0)(x - x_1)\cdots(x - x_{n-1})a_n \end{aligned}$$

Coefficients:

$$a_0 = y_0,\ a_1 = \nabla y_1,\ a_2 = \nabla^2 y_2,\ \cdots,\ a_n = \nabla^n y_n$$

Divided differences:

$$\nabla y_i = \frac{y_i - y_0}{x_i - x_0}, \quad i = 1, 2, \ldots, n$$

$$\nabla^2 y_i = \frac{\nabla y_i - \nabla y_1}{x_i - x_1}, \quad i = 2, 3, \ldots, n$$

$$\nabla^3 y_i = \frac{\nabla^2 y_i - \nabla^2 y_2}{x_i - x_2}, \quad i = 3, 4, \ldots n$$

$$\vdots$$

$$\nabla^n y_n = \frac{\nabla^{n-1} y_n - \nabla^{n-1} y_{n-1}}{x_n - x_{n-1}}$$

x_0	y_0				
x_1	y_1	∇y_1			
x_2	y_2	∇y_2	$\nabla^2 y_2$		
x_3	y_3	∇y_3	$\nabla^2 y_3$	$\nabla^3 y_3$	
x_4	y_4	∇y_4	$\nabla^2 y_4$	$\nabla^3 y_4$	$\nabla^4 y_4$

Newton's Method

Matlab Implementation

```
function p = fun_newton_interp(xd, yd, x)
%Interpolation function using Newton method
%   Description: This function performs interpolation using Newton method.
%
%   Usage      : p = fun_newton_interp(xd, yd, x)
%
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m = length(xd);                          % number of data

% Calculating coefficients a
a = yd;
for k = 2 : m
    for i = k : m
        a(i) = (a(i) - a(k-1))/(xd(i) - xd(k-1));
    end
end

% function evaluation
p = a(m)*ones(length(x),1);
for k = m - 1 : -1 : 1
    p = a(k) + (x - xd(k)).*p;
end
```